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THE INTERPRETATION OF NONLINEAR PITCHING MOMENTS IN RELATION TO THE PITCH-UP PROBLEM

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THE INTERPRETATION OF NONLINEAR PITCHING MOMENTS IN RELATION TO THE PITCH-UP PROBLEM[1]

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SUMMARY

Practical methods are presented for predicting the longitudinal response of an airplane to arbitrary control inputs using nonlinear aerodynamic data. These methods are used to study the pitch-up problem encountered by numerous present-day airplanes in maneuvering flight at high speeds.

Of the variables affecting airplane longitudinal response characteristics, pitching-moment variations with angle of attack and with Mach number are of primary importance. The consideration of control movements by the pilot is important for the milder pitching-moment nonlinearities, but with a severe instability, pitch-up motion is little affected by control movements. For a given shape of the pitching-moment curve, the severity of pitch-up is increased by either an increase in dynamic pressure or a decrease in the airplane longitudinal moment of inertia.

In the event that pitch-up cannot be eliminated through geometric modification to a particular configuration, automatic stabilization devices may offer a means of improving marginal flight behavior.

INTRODUCTION

A large number of present-day high-speed airplanes encounter a longitudinal instability at moderately high lift coefficients that is commonly referred to as "pitch-up." The sudden and often uncontrollable increase in angle of attack and normal acceleration characterizing this type of instability is always undesirable from the standpoint of the pilot even when critical loads are not exceeded.

In order to determine the true significance of aerodynamic nonlinearities obtained during wind-tunnel investigations, it is necessary to have a method for converting static nonlinear data into time histories

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[1]Supersedes declassified NACA Research Memorandum L53I02 by George S. Campbell and Joseph Weil, 1953.

of airplane motions. Inasmuch as available methods (refs. 1 and 2) are not sufficiently general to permit consideration of the effects of arbitrary control motions or the variation of aerodynamic characteristics with Mach number, this report presents methods of analysis suitable for a detailed treatment of the pitch-up problem. Application of the methods derived is directed toward a study of some of the factors affecting pitch-up behavior, such as shape of pitching-moment curve, control movement, dynamic pressure, inertia effects, and aerodynamic damping. Brief consideration is also given to the effectiveness of automatic stabilization devices in reducing pitch-up severity.

SYMBOLS

The system of axes used throughout the present report is illustrated in figure 1 along with the directions for positive forces, moments, and angles. All angles are measured in radians unless specifically noted otherwise. Differentiation with respect to time has been designated by means of a dot (or dots) above the dependent variable.

X	wind axis tangent to flight path
Z	wind axis normal to flight path
L	lift, lb
D	drag, lb
M_{cg}	pitching moment about airplane center of gravity, ft-lb
W	airplane weight, lb
m	airplane mass, slugs
T	airplane thrust (assumed to act through center of gravity), lb
I_y	longitudinal moment of inertia about airplane center of gravity, slug-ft^2
V	forward velocity, ft/sec
t	time, sec
α	angle of attack, measured from thrust axis
γ	flight-path angle

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θ	airplane attitude, $\alpha + \gamma$
C_L	lift coefficient, $2L/\rho V^2 S$
C_D	drag coefficient, $2D/\rho V^2 S$
C_m	moment coefficient, $2M_{cg}/\rho V^2 S\bar{c}$
W'	dimensionless weight parameter, $2W/\rho {V_1}^2 S$
μ	relative density, $m/\rho S\bar{c}$
T'	dimensionless thrust parameter, $2T/\rho {V_1}^2 S$
u	dimensionless velocity, V/V_1
M	Mach number, V/a
a	speed of sound in air, ft/sec
τ	airplane time factor, $m/\rho SV_1$, sec
ν	dynamic-response parameter, $\dfrac{\rho {V_1}^2 S\bar{c}}{2I_y}$, $\dfrac{\text{radians}}{\text{sec}^2}$
ρ	density of air, slugs/ft^3
S	wing area, ft^2
$\bar{c}$	mean aerodynamic chord, ft
i_t	stabilizer deflection
δ_e	elevator deflection
C_{m_q}	damping derivative, $\dfrac{\partial C_m}{\partial q\bar{c}/2V}$
$C_{m_{\dot{\alpha}}}$	damping derivative, $\dfrac{\partial C_m}{\partial \dot{\alpha}\bar{c}/2V}$
q	pitching velocity, $d\theta/dt$
n	normal-load factor (ratio of the aerodynamic force normal to the angle-of-attack reference to the airplane weight)

K_y dimensionless radius of gyration, $\sqrt{I_y/m\bar{c}^2}$

ϵ downwash angle

b damping parameter, equation (8), $\sec^{-1}$

k dimensionless restoring moment, equation (9)

C_{L_α} local slope of lift curve, $dC_{L_o}/d\alpha$

$i_t{}^*$ autopilot contribution to control deflection

h_p altitude, ft

$C_{D_{min}}$ minimum drag coefficient

ΔC_D drag due to lift

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Subscripts:

1 initial value

o curve defining static variation of coefficients C_m, C_L, and C_D when controls are fixed in their initial positions i_{t_1}, δ_{e_1}

i_t partial derivative with respect to i_t

δ_e partial derivative with respect to δ_e

DERIVATION OF METHOD

Basic Equations of Motion

Application of Newton's laws provides the basic longitudinal equations of motion of an airplane having three degrees of freedom and a system of wind axes (fig. 1):

$$\left.\begin{aligned} -W \sin \gamma - D + T \cos \alpha &= m \frac{dV}{dt} \\ W \cos \gamma - L - T \sin \alpha &= -mV \frac{d\gamma}{dt} \\ M_{cg} &= I_y \frac{d^2\theta}{dt^2} \end{aligned}\right\} \tag{1}$$

After rewriting equations (1) in terms of convenient parameters defined in the section entitled "Symbols," the equations become

$$\left.\begin{aligned} -W' \sin\gamma - u^2 C_D + T' \cos\alpha &= 2\tau\dot{u} \\ W' \cos\gamma - u^2 C_L - T' \sin\alpha &= -2\tau u\dot{\gamma} \\ \nu u^2 C_m &= \ddot{\gamma} + \ddot{\alpha} \end{aligned}\right\} \tag{2}$$

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By treating ν and τ as constants, the assumption is implicitly made that changes in mass, inertia, and air density may be neglected during the maneuver.

For calculation purposes, it is necessary to expand the aerodynamic coefficients C_D, C_L, and C_m into terms representing the separate effects of static force and moment characteristics, control-surface deflections, and airplane damping:

$$C_D = C_{D_o} + C_{D_{i_t}}\left(i_t - i_{t_1}\right) + C_{D_{\delta_e}}\left(\delta_e - \delta_{e_1}\right) \tag{3a}$$

$$C_L = C_{L_o} + C_{L_{i_t}}\left(i_t - i_{t_1}\right) + C_{L_{\delta_e}}\left(\delta_e - \delta_{e_1}\right) \tag{3b}$$

$$C_m = C_{m_o} + C_{m_{i_t}}\left(i_t - i_{t_1}\right) + C_{m_{\delta_e}}\left(\delta_e - \delta_{e_1}\right) + \frac{\tau}{2\mu u}\left(C_{m_q}\dot{\theta} + C_{m_{\dot{\alpha}}}\dot{\alpha}\right) \tag{3c}$$

Whenever necessary, additional terms may be added to equations (3) to account for such effects as lift and drag due to pitching and plunging motions, nonlinear variations of C_m with control movement, and so forth. The stability derivatives appearing in equations (3) may have arbitrary variations with α and M, and the control inputs i_t and δ_e are functions of the variables which influence a pilot's response, such as n, α, M, t, and so forth.

After substituting equations (3) into (2), the set of three simultaneous differential equations for the unknowns α, γ, and u takes the form

$$-W' \sin \gamma - u^2\left[C_{D_o} + C_{D_{i_t}}\left(i_t - i_{t_1}\right) + C_{D_{\delta_e}}\left(\delta_e - \delta_{e_1}\right)\right] + T' \cos \alpha = 2\tau\dot{u} \tag{4a}$$

$$W' \cos \gamma - u^2\left[C_{L_o} + C_{L_{i_t}}\left(i_t - i_{t_1}\right) + C_{L_{\delta_e}}\left(\delta_e - \delta_{e_1}\right)\right] - T' \sin \alpha = -2\tau u\dot{\gamma} \tag{4b}$$

$$\nu u^2\left[C_{m_o} + C_{m_{i_t}}\left(i_t - i_{t_1}\right) + C_{m_{\delta_e}}\left(\delta_e - \delta_{e_1}\right)\right] + \frac{u}{4\tau K_y{}^2}\left[C_{m_q}\dot{\gamma} + \left(C_{m_q} + C_{m_{\dot{\alpha}}}\right)\dot{\alpha}\right] = \ddot{\gamma} + \ddot{\alpha} \tag{4c}$$

Equations (4) may be solved by using numerical procedures such as the Runge-Kutta method described in reference 3. Having solved for α and u, the normal-load factor may also be calculated by using the relation

$$n = \frac{\left(C_L \cos \alpha + C_D \sin \alpha\right)u^2}{W'} \tag{5}$$

where C_L and C_D are given by equations (3b) and (3a), respectively.

Experimental values of the damping derivatives required in equations (4) are seldom available without limitations imposed by either low speeds (refs. 4 to 6) or low lift coefficients (refs. 7 and 8). For airplane configurations having horizontal tails, however, the damping derivatives at high lift can be estimated by using static wind-tunnel measurements of $C_{m_{i_t}}$ and $d\epsilon/d\alpha$, as pointed out in reference 9. For tailless configurations, it may be necessary to use low-lift damping derivatives until experiment provides more reliable information.

Simplified Equations of Motion

Even with the use of present-day high-speed digital computers, the time involved in solving equations (4) may be excessive. Moreover, the mathematics of the problem must be considerably simplified if it is to lead to a physical understanding of pitch-up.

The basic approach in the simplified calculation method is to obtain solutions for α and γ from a two-degree-of-freedom system involving approximate forms of equations (4b) and (4c). The remaining equation (4a) is then used to provide an approximate value of the speed variation $u(t)$.

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A major assumption underlying derivation of the equations describing two-degree-of-freedom motion is that the change in speed during the maneuver may be neglected. In addition, the lift equation (4b) may be further simplified if the tail lift and the Z-component of airplane thrust are neglected and if the flight path angle is sufficiently small so that $\cos\gamma$ may be taken as unity. With these assumptions, the equations of motion for two degrees of freedom become

$$C_{L_o} - W' = 2\tau\dot{\gamma} \tag{6a}$$

$$\nu\left[C_{m_o} + C_{m_{i_t}}\left(i_t - i_{t_1}\right) + C_{m_{\delta_e}}\left(\delta_e - \delta_{e_1}\right)\right] + \frac{1}{4\tau K_y^2}\left[C_{m_q}\dot{\gamma} + \left(C_{m_q} + C_{m_{\dot{\alpha}}}\right)\dot{\alpha}\right] = \ddot{\gamma} + \ddot{\alpha} \tag{6b}$$

where the aerodynamic parameters are now considered functions of α only.

The first of these expressions may be substituted into the second so as to eliminate γ and provide a single equation for angle of attack

$$\ddot{\alpha} + b\dot{\alpha} + \nu k = \nu\left[C_{m_{i_t}}\left(i_t - i_{t_1}\right) + C_{m_{\delta_e}}\left(\delta_e - \delta_{e_1}\right)\right] \tag{7}$$

in which

$$b = \frac{1}{2\tau}\left[C_{L_\alpha} - \frac{1}{2K_y^2}\left(C_{m_q} + C_{m_{\dot{\alpha}}}\right)\right] \tag{8}$$

$$k = -C_{m_o} - \frac{C_{m_q}\left(C_{L_o} - W'\right)}{4\mu} \tag{9}$$

Although the two terms comprising the damping coefficient b are generally of comparable magnitude, the second term in the expression for k may in most cases be neglected so that equation (7) becomes

$$\ddot{\alpha} + b\dot{\alpha} - \nu C_{m_o} = \nu\left[C_{m_{i_t}}\left(i_t - i_{t_1}\right) + C_{m_{\delta_e}}\left(\delta_e - \delta_{e_1}\right)\right] \tag{10}$$

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Once the time history of α has been calculated from equation (10) (by using the numerical method of Runge-Kutta, for example), pitching velocity and flight-path angle may be obtained from approximate formulas derived from equation (6a):

$$q = \frac{1}{2\tau}\left(C_{L_o} - W'\right) + \dot{\alpha} \tag{11}$$

$$\gamma = \gamma_1 + \frac{1}{2\tau}\int_0^t \left(C_{L_o} - W'\right)dt \tag{12}$$

Although, if necessary, equation (4a) may be integrated numerically in order to provide the speed variation during a maneuver, an approximate form of this equation

$$u^2 C_{D_o} = -2\tau\dot{u} \tag{13}$$

may be integrated by simple quadrature

$$u = \frac{1}{1 + \frac{1}{2\tau}\int_0^t C_{D_o}\,dt} \tag{14}$$

Equation (14) has been found useful for calculating the speed loss during pitch-ups at moderately high speed, in which the neglected terms are of secondary importance.

Inasmuch as equation (5) for normal load factor requires no simplification, approximate relations have been developed for all parameters obtainable from the more complicated equations of motion.

RESULTS AND DISCUSSION

Reliability of Calculation Methods

Comparison between basic and simplified methods.- Two methods for calculating the longitudinal response of an airplane to arbitrary control motions have been derived in the previous section. (See eqs. (4) and (10).) The simpler method, equation (10), was developed primarily for studies in which the aerodynamic characteristics can be considered invariant with Mach number. In order to determine whether the simpler method provides results in agreement with the more exact solution, time-history calculations of identical pull-up maneuvers have been made for a hypothetical airplane configuration having a region of longitudinal instability at moderately high lift coefficients. The aerodynamic characteristics used in the calculations are summarized in figure 2, and the results of the longitudinal response calculations for a ramp-type-stabilizer motion are presented in figure 3.

It is seen that the simplified method provided results that are in good agreement with the more exact method. Not only does the more approximate method reduce the computing time by a factor of 3 or more, but it also facilitates understanding of the relative importance of the various parameters affecting the motion.

In addition to providing a comparison of results obtained by using the simplified and more refined methods of calculation, figure 3 illustrates the typical behavior of airplanes having pitch-up. The most apparent characteristic of the pitch-up motion is the sudden increase in angle of attack occurring near the angle at which the static pitching-moment curve becomes unstable. The increase in normal acceleration accompanying the change in α is softened as a result of the reduction of lift-curve slope with angle of attack. A further reduction in normal acceleration is produced by the loss of forward speed during the maneuver. However, no matter how gradual the build-up of normal acceleration during pitch-up, pilots object to a rapid change of airplane attitude, especially if the change is uncontrollable.

Comparisons between calculated and flight-test results.- A number of comparisons between calculated and flight-measured time histories are shown in reference 10 for the Bell X-5 airplane. The simplified equations of the present paper were used in the calculations together with the measured wind-tunnel results. Satisfactory agreement was shown between the computed and flight-test results for the various parameters defining the motion (see ref. 10).

Study of Some of the Factors Affecting Pitch-Up

Control motion.- The effects of varying initial rate of control deflection are presented in figure 4 for an airplane configuration having unstable pitching-moment characteristics in the angle-of-attack range between 8° and 16°. (See table I for other parameters used in calculations.) It is evident from the results that for a given change in absolute control deflection there was practically no effect of varying the rate of control application on the maximum values of either α or $\dot{\alpha}$. For maneuvers in which control motion ceased at a given angle of attack ($\alpha = 14^{\circ}$), however, the peak values of α and $\dot{\alpha}$ were somewhat higher for the more rapid maneuvers as a result of the larger final control deflection. The time histories in the rest of this report have been calculated for gradual pull-up maneuvers in which the initial stabilizer rate is 1 or 2 degrees per second.

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Another point of interest concerns the pilot's ability to arrest the motion of the airplane once aware of the onset of pitch-up. The critical dependence of the amount of overshoot on the point at which corrective control is applied is illustrated in figure 5 where the vertical ticks on the response curves indicate the start of corrective control. For an airplane having the pitching-moment characteristics assumed, it would appear that if corrective control were deferred until pitch-up was apparent, there would be little chance of avoiding a large overshoot. In order to prevent large overshoot for such a severe instability, corrective control would have to be applied at or before the angle of attack for which the pitching moment becomes unstable - which is very unlikely unless a warning such as the onset of buffeting is given slightly in advance of pitch-up.

The effect of varying the rate of corrective control is shown in figure 6. For these maneuvers the pilot was assumed to apply corrective control about 1 second after initiation of pitch-up. The results indicate that only a very rapid rate of corrective control reduced the peak angle by an appreciable amount. Note also that with corrective control the airplane enters the unstable pitching-moment region from the opposite direction and executes an abrupt "pitch down." One such motion encountered in flight was of sufficient severity to cause the airplane to reach -3g. (See ref. 11).

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Shape of the pitching-moment curve $C_m = f(\alpha)$.- The results of dynamic-response calculations made to investigate the influence of steepness and extent of static instability on the pitch-up behavior of an airplane are presented in figure 7. A ramp-type control motion was used for the calculations, the stabilizer variation with time being identical for all cases. Additional parameters used in the calculations are presented in table I. The time histories of figure 7 show the expected increase in severity of pitch-up with increased steepness and extent of instability. A mild but broad static instability is seen to be equivalent to a steeper instability of less breadth.

In order to investigate the interrelation between the shape of the pitching-moment curve and corrective control, the time histories of figure 8 were calculated by using the three variations in pitching-moment shape shown in the figure. The vertical ticks on the response curve indicate the point at which the pilot attempted to arrest the motion either by holding the control fixed or by applying corrective stabilizer at a rate of 4 degrees per second. It is seen from figure 8 that although corrective control had little effect on the severity of pitch-up for the pitching moment with a region of pronounced instability (curve A), the same rate of control movement effected a marked reduction in the maximum angle of attack for the milder instability (curve B). However, curve B cannot be considered satisfactory since an airplane with such pitching moments would require the constant attention of the pilot to prevent inadvertently reaching high angles of attack during maneuvers.

Dynamic response parameter ν.- Inasmuch as both static pitching moments and moments due to control input are multiplied by the factor $\nu = \frac{\rho V_1^2 S\bar{c}}{2I_Y}$ in equation (10), it would appear that this parameter could have an important bearing on the pitch-up motion of an airplane. In order to verify the importance of the response parameter ν, the time histories of figure 9 were calculated by using a pitching-moment curve having a region of neutral stability at moderate angles of attack. It was assumed that the pilot, in making a gradual pull-up, desired to arrest the airplane motion at an angle of attack of about 8°. However, because of control lag and reaction-time delay, it was further assumed that 1/2 second elapsed before the control motion was either stopped or reversed. The results of the time-history calculations indicate that for a response factor ν of 16 (representative of a fighter-type airplane loaded primarily along the fuselage and flying at altitudes of 30,000 to 40,000 feet at transonic speed), the application of corrective control caused an appreciable reduction in the amount of overshoot. For a value of $\nu = 64$ (representative of a fighter-type airplane loaded primarily along the wing), the motion built up so rapidly that corrective control was completely ineffective in reducing the peak angle attained.

Damping parameter b.- The effect of a reduction in damping on the motion of a configuration having the same pitching-moment curve as used in figure 9 is presented in figure 10. Damping A was used in previous calculations and is representative of the damping for an airplane with a horizontal tail. Damping B represents only the wing contribution $C_{L_{\alpha}}$ to the damping and is therefore somewhat representative of a tailless airplane.

The results of the calculations show that a marked reduction in damping only slightly increased the maximum rate $\dot{\alpha}$ and the peak angle attained during the pitch-up. The general character of the response curve was little affected by the change in damping considered in this calculation.

Shape of the pitching-moment curve $C_m = f(\alpha,M)$.- In previous paragraphs, the importance of shape of the pitching-moment curve was discussed for cases in which C_m was dependent only upon angle of attack. In certain instances, however, the airplane pitching-moment characteristics may be significantly affected by changes in Mach number during a maneuver as well as by angle of attack. For example, during a maneuver at transonic speeds, the airplane may slow down rapidly enough so that a forward shift in aerodynamic-center position due to Mach number change can cause pitch-up. In order to investigate the importance of this type of pitch-up, the pitching moments shown in figure 11 were used to calculate the time histories presented in figure 12; the additional parameters for these calculations are given in table II. For case A (fig. 11), linear pitching-moment curves were assumed along with an aerodynamic-center shift consistent with representative experimental results. For case B, this same type of aerodynamic-center shift was superimposed on a nonlinear variation of C_{m_0} with α. Using these pitching-moments, time histories of angle of attack and Mach number (fig. 12) were calculated for a stabilizer input in which the pilot was assumed to stop moving the control when he reached 11.5° angle of attack.

With linear pitching-moment curves (case A), the angle of attack response at first tended to follow the control motion (in the vicinity of $t = 4$ seconds), but the changing moment characteristics accompanying the decrease in Mach number finally resulted in a divergence in angle of attack. However, the divergence built up rather slowly so that there was less than 2° overshoot after the application of corrective control. The controllability of this type of pitch-up is probably critically dependent upon the dynamic-response parameter $\nu = \dfrac{\rho V_1{}^2 S\bar{c}}{2I_Y}$ which was $19.2\ \dfrac{\text{radians}}{\text{sec}^2}$ for the results of figure 12. For case B, the pitch-up was more abrupt than for case A as a result of the nonlinearity of C_{m_0} with angle of attack.

In more general terms, the results indicate the importance of evaluating pitching moments not only on the basis of their variations with angle of attack but also from a consideration of changes in aerodynamic-center position, trim, and overall shape of the pitching-moment curve with Mach number. What might appear to be a mild nonlinearity at a constant Mach number could result in severe pitch-up when the airplane undergoes rapid speed changes.

Automatic Stabilization Devices

Although certainly desirable, it may not always be practical to correct an airplane pitch-up tendency through geometric modification. In this eventuality, the use of an autopilot may offer a means of obtaining acceptable maneuvering characteristics.

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The manner in which an autopilot changes stability is shown by the following equation:

$$\ddot{\alpha} + b\dot{\alpha} - \nu C_{m_o} = \nu C_{m_{i_t}}\left[\left(i_t - i_{t_1}\right) + i_t^*\right] \tag{15}$$

where $\left(i_t - i_{t_1}\right)$ is the control contributed by the pilot and i_t^* by the autopilot. The last term on the right side of equation (15) is used to feed artificial stability into the system.

The autopilot contribution required for a linear angle-of-attack response ($\dot{\alpha}$ = Constant) may be calculated by substituting the desired response

$$\alpha = \alpha_1 + \dot{\alpha}t \tag{16}$$

into equation (15) and solving for i_t^*

$$i_t^* = \frac{b\dot{\alpha}}{\nu C_{m_{i_t}}} - \frac{C_{m_o}}{C_{m_{i_t}}} - \left(i_t - i_{t_1}\right) \tag{17}$$

In figure 13, results of calculations using equations (16) and (17) are presented for an airplane having a static instability in the moderate angle-of-attack range. The autopilot contribution i_t^* necessary to produce a linear angle-of-attack response for a linear control input by the pilot is seen to have a gradual variation with time.

The variation of i_t^* with α obtained from the idealized autopilot calculation (fig. 13) suggests the possibility of using an autopilot sensitive to angle of attack to eliminate pitch-up. Calculated maneuvering characteristics of an airplane incorporating this type of autopilot are presented in figure 14 and compared with the motion of the basic airplane. For these calculations, it was assumed that the autopilot contribution i_t^* varied linearly with angle of attack, and that this contribution was simply superimposed on the pilot's arbitrary control movement. The results show that with the autopilot operating, the airplane followed the pilot's control movement with no indication of pitch-up.

Although the calculations show that an autopilot could be expected to correct pitch-up even for a severe static instability, the primary value of such a device might well be that of improving the response characteristics of an airplane having a mild form of pitch-up. Then, failure of the automatic stabilization system would not lead to destruction of the airplane. L-795

There has been no attempt in the present investigation to make a detailed study of automatic stabilization systems. Other types of autopilots (such as devices sensitive to pitching velocity) or combinations of types may offer possible means of controlling pitch-up.

CONCLUSIONS

Methods have been derived by which time histories of longitudinal motions can be calculated for configurations having arbitrary nonlinear pitching-moment characteristics and control inputs. Good correlation has been obtained between time histories predicted using the methods of this report and those measured in flight for an airplane having pitch-up. From a study of the pitch-up problem using the methods derived, the following conclusions may be made:

1. Of the factors affecting the pitch-up motion of an airplane, shape of the static pitching-moment curve is of primary importance. Once an airplane enters a region of pronounced instability, the airplane pitch-up motion is, in general, little affected by control movements by the pilot.

2. The amount of control which a pilot has over airplane motion during pitch-up is strongly dependent upon the dynamic-response parameter $\nu = \frac{\rho V_1{}^2 S\bar{c}}{2I_Y}$, which represents the ratio of aerodynamic moments to

the airplane moment of inertia. For a given shape of pitching-moment curve, an increase in the value of ν reduces the controllability of the pitch-up.

3. If pitching-moment variations with Mach number are sufficiently abrupt, pitch-up can result even in the absence of nonlinearities with angle of attack.

4. In the event that satisfactory pitching-moment characteristics cannot be obtained from geometric modifications to a configuration, the use of automatic stabilization devices offers a possible means of controlling pitch-up.

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Langley Research Center,
National Aeronautics and Space Administration,
Langley Field, Va., August 25, 1953.

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TABLE I

STANDARD SET OF PARAMETERS USED FOR TIME HISTORY CALCULATIONS

[Except where otherwise noted, the time histories in this report were calculated by using the simplified equation of motion (10) and the following standard set of parameters.]

α, deg	b, sec^{-1}
0-14	1.71
16	1.45
18	1.11
20	.91
24	.64
28	.45
32	.29
36	.16

M_1	0.85
h_p, ft	30,000
W/S, lb/sq ft . . .	70
K_Y	0.9
$\bar{c}$, ft	11.3
$C_{m_{i_t}}$, deg^{-1}	-0.02
ν, radians/sec^2 . .	16.0

TABLE II

PARAMETERS USED FOR CALCULATING TIME HISTORIES OF FIGURE 12

$$\left[C_{D_o} = C_{D_{min}} + \Delta C_D ;\right.$$

$$\left.C_{D_{min}} = 0.015 + 0.125(M - 0.9);\ 0.9 \leqq M \leqq 1.1\right]$$

α, deg	C_{L_o}	ΔC_D	$C_{m_q} + C_{m_{\dot{\alpha}}}$ radians^{-1}
0 . .	0	0	-8.80
4 . .	0.24	0.015	-8.80
8 . .	0.48	0.060	-8.80
12 . .	0.72	0.136	-8.80
16 . .	0.92	0.231	-6.96
20 . .	1.04	0.326	-5.87
24 . .	1.08	0.407	-3.30
28 . .	1.08	0.474	-3.30
32 . .	1.08	0.543	-3.30

M_1	1.1
h_p, ft	30,000
W/S, lb/sq ft	80
T/S, lb/sq ft	24.4
K_y	1
$\bar{c}$, ft	11.17
$C_{m_{i_t}}$, deg^{-1}	-0.02
$C_{L_{i_t}}$, deg^{-1}	0.01
$C_{D_{i_t}}$	0
C_{m_q}, radians^{-1} . . .	-5.5
ν, radians/sec^2 . . .	19.2

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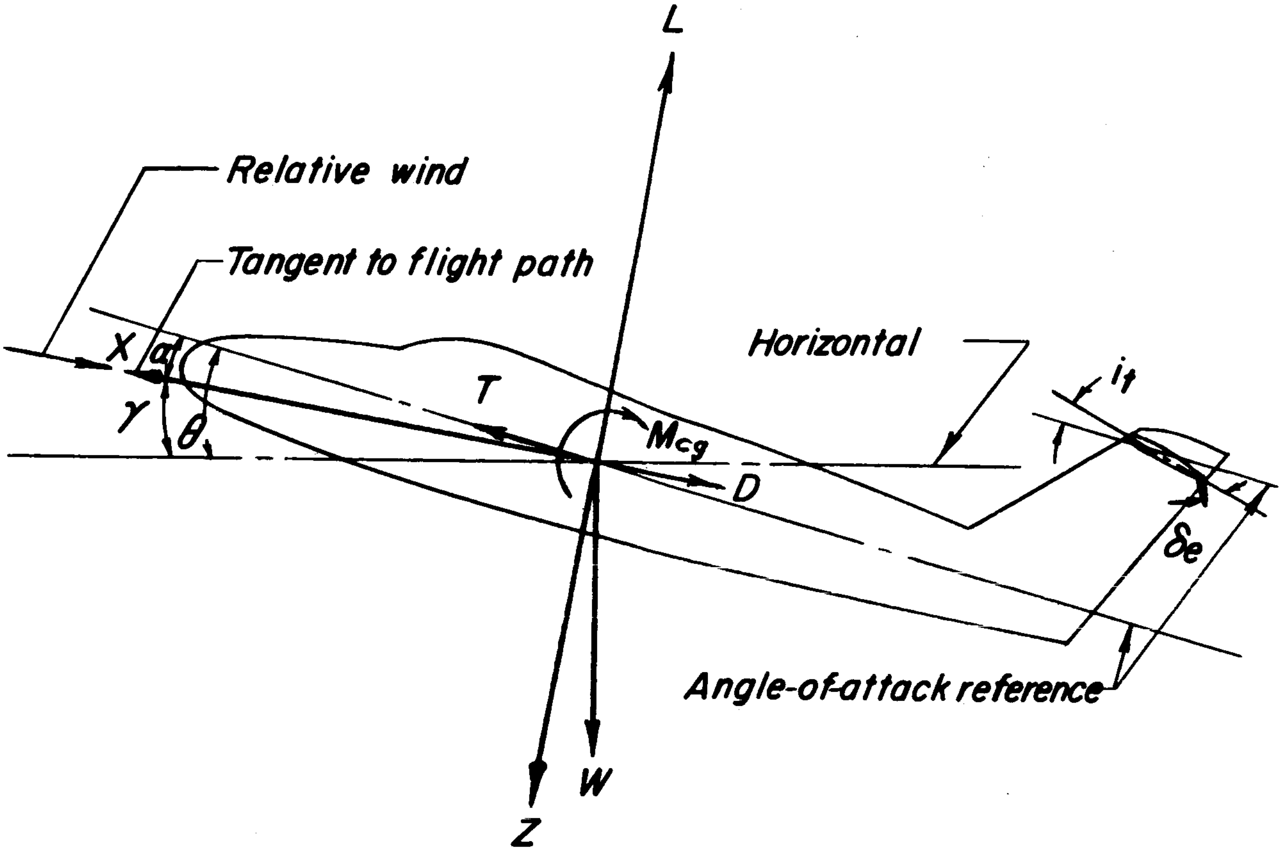
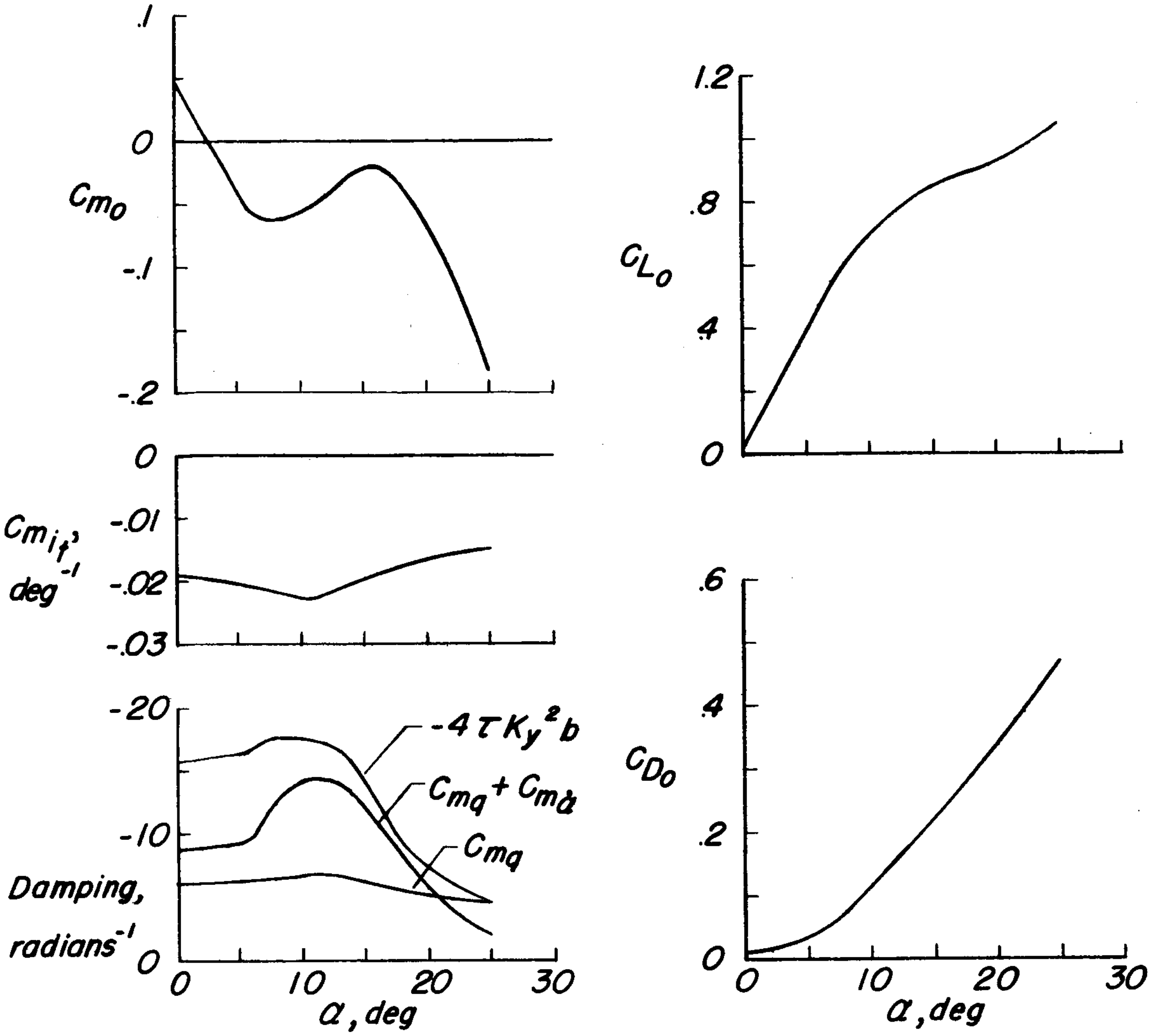


Figure 1.- System of wind axes and directions for positive forces, moments, and angles.



M_0	0.85	$\bar{c}$, ft	11.3
h_p, ft	30,000	$C_{L_{i_t}}$, radians^{-1}	0.573
W/S, lb/sq ft	70	$C_{D_{i_t}}$	0
T/S, lb/sq ft	4.77	δ_e	0
K_y	0.9		

Figure 2.- Aerodynamic characteristics and arbitrary parameters used for calculating the time histories of figure 3.

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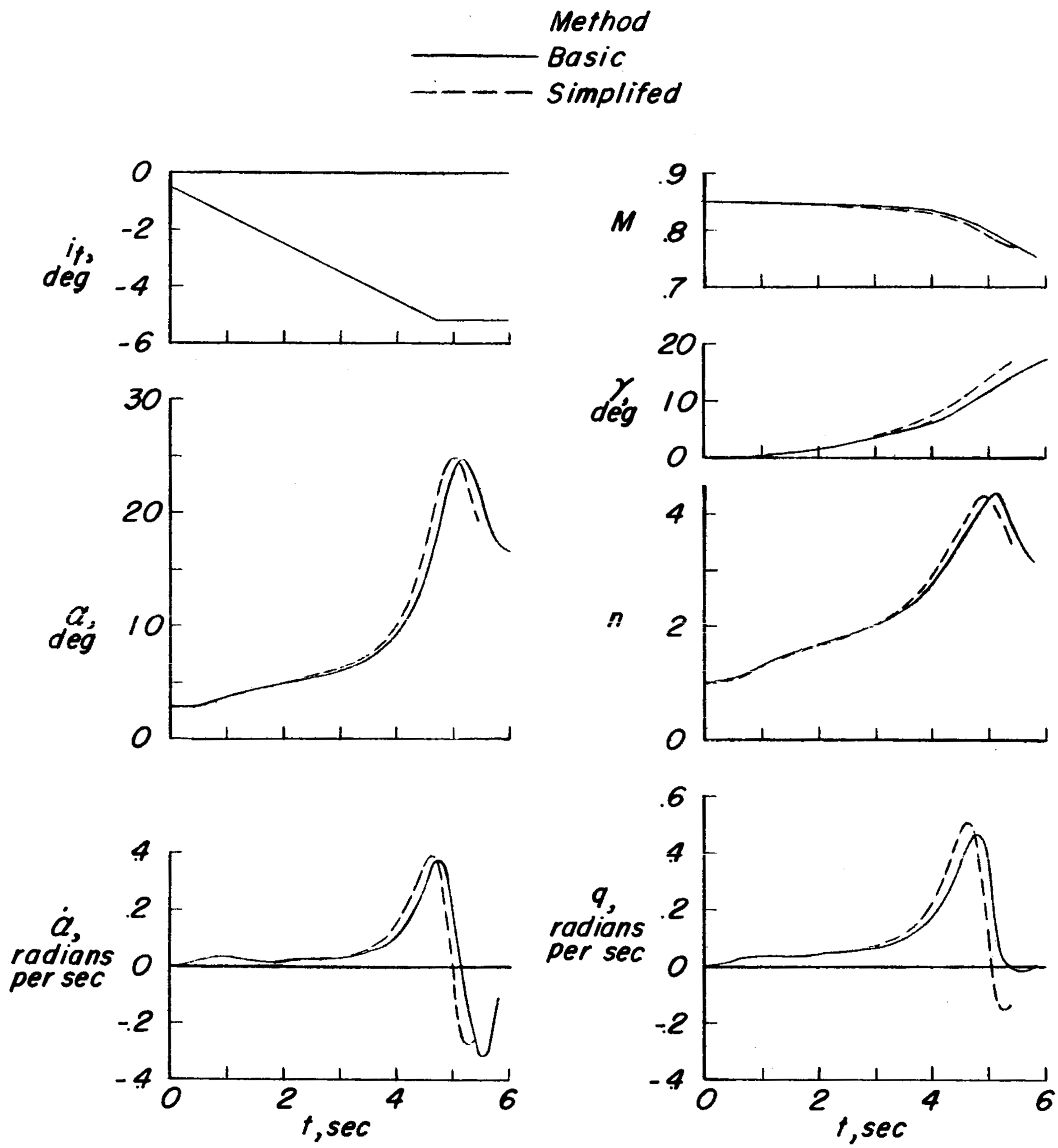

Figure 3.- Comparison of time histories calculated by using basic and simplified equations of motion.

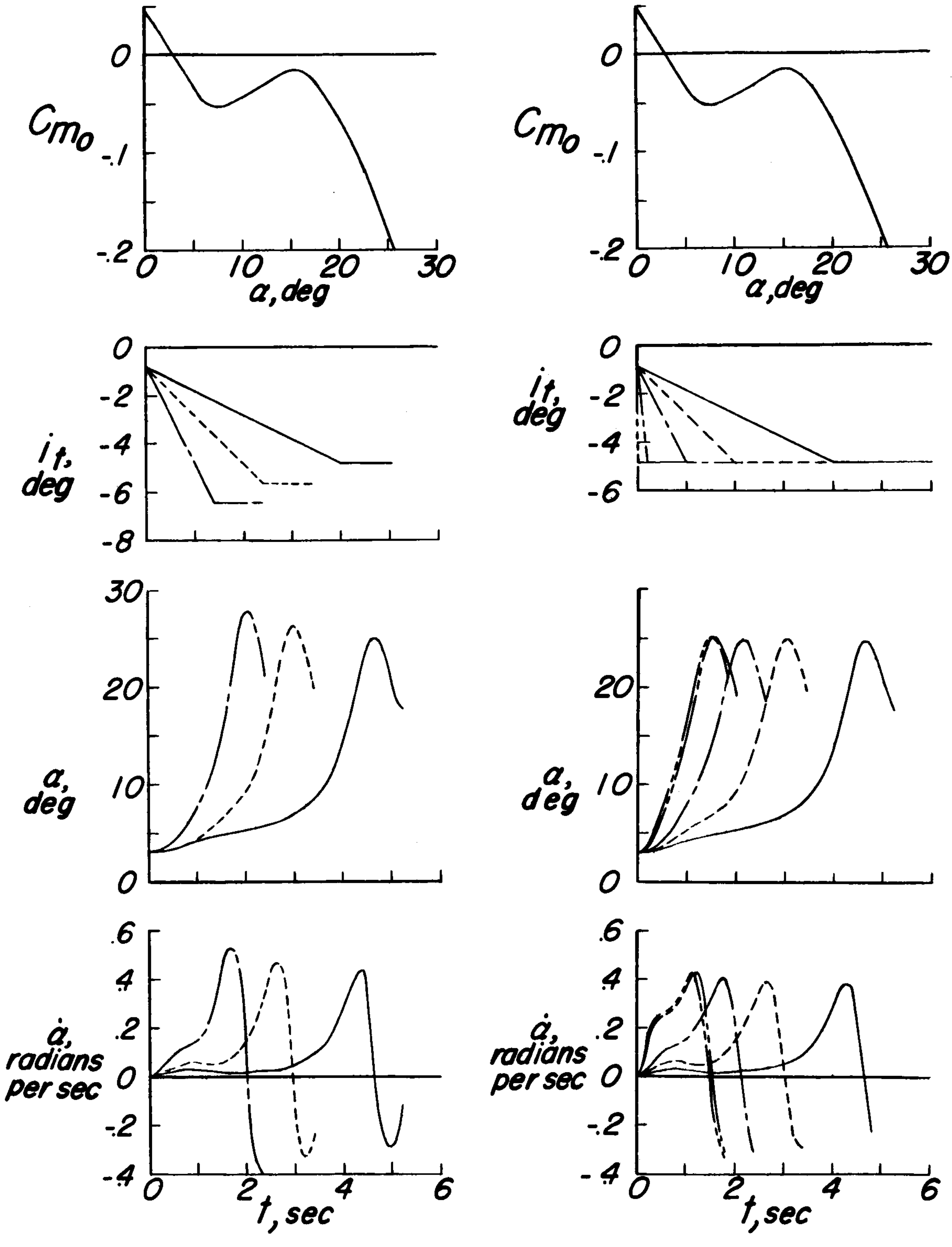

Figure 4.- Effect of initial rate of control deflection on pitch-up behavior.

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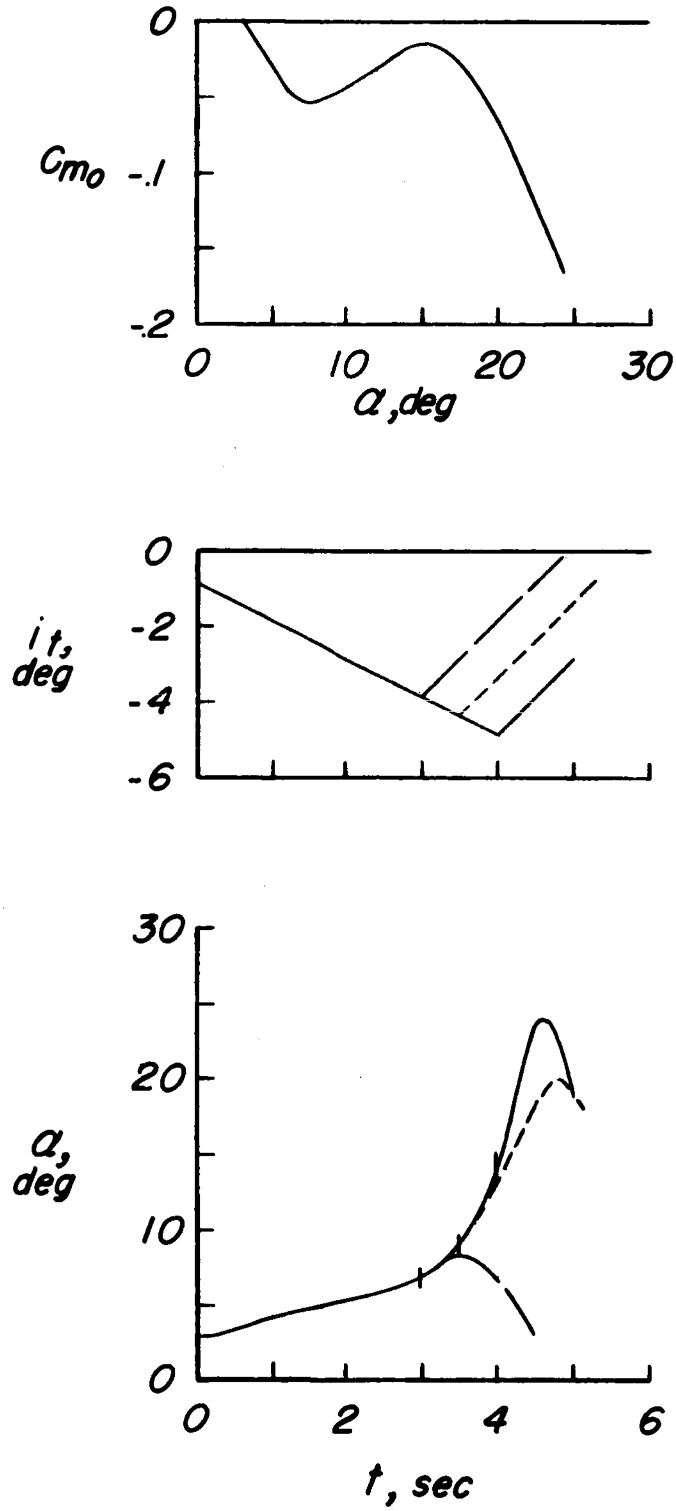

Figure 5.- Effect on pitch-up of the time at which corrective control is applied.

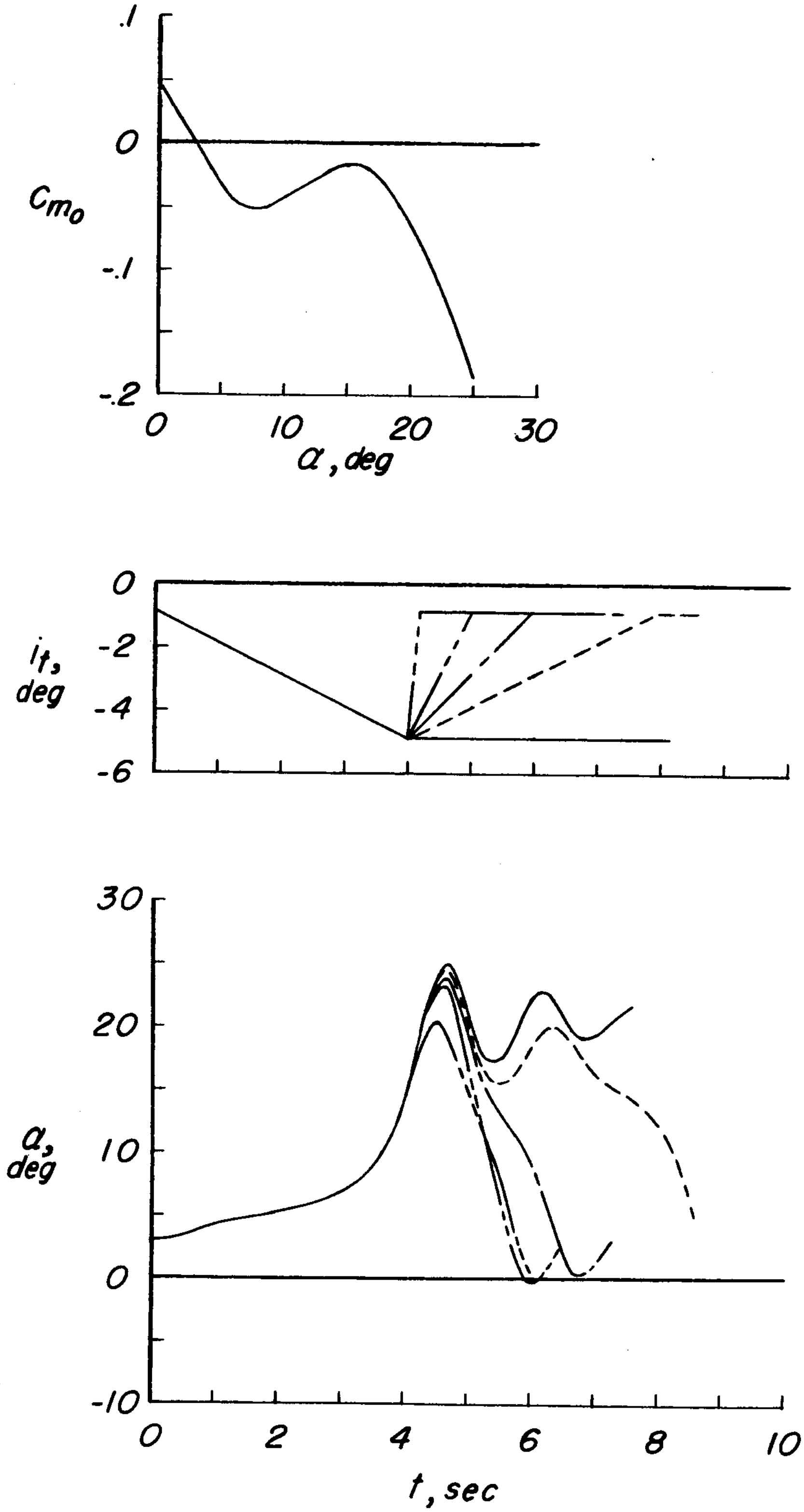

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Figure 6.- Effect on pitch-up of the rate of corrective control application.

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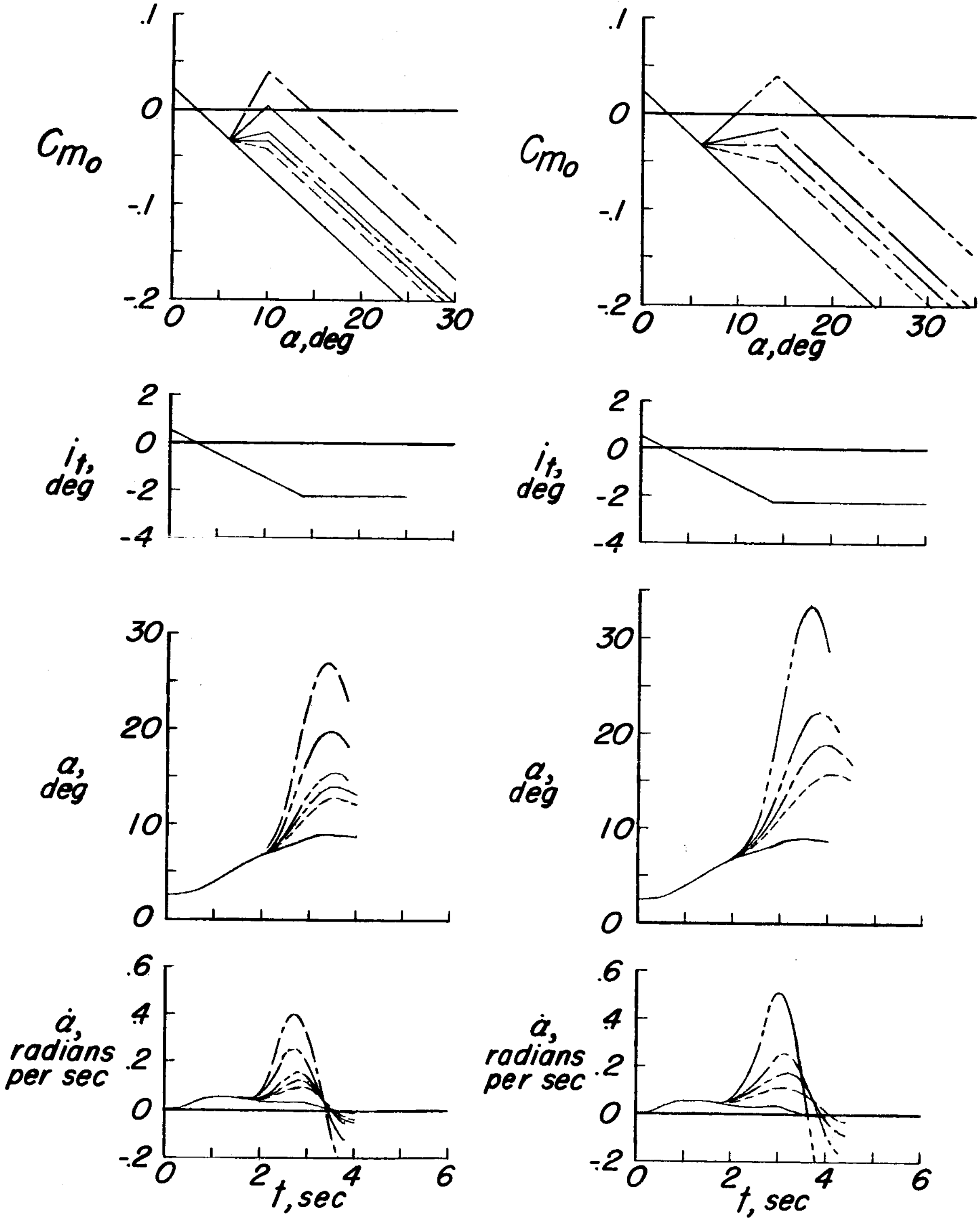

Figure 7.- Effect of steepness and extent of static instability on pitch-up behavior.

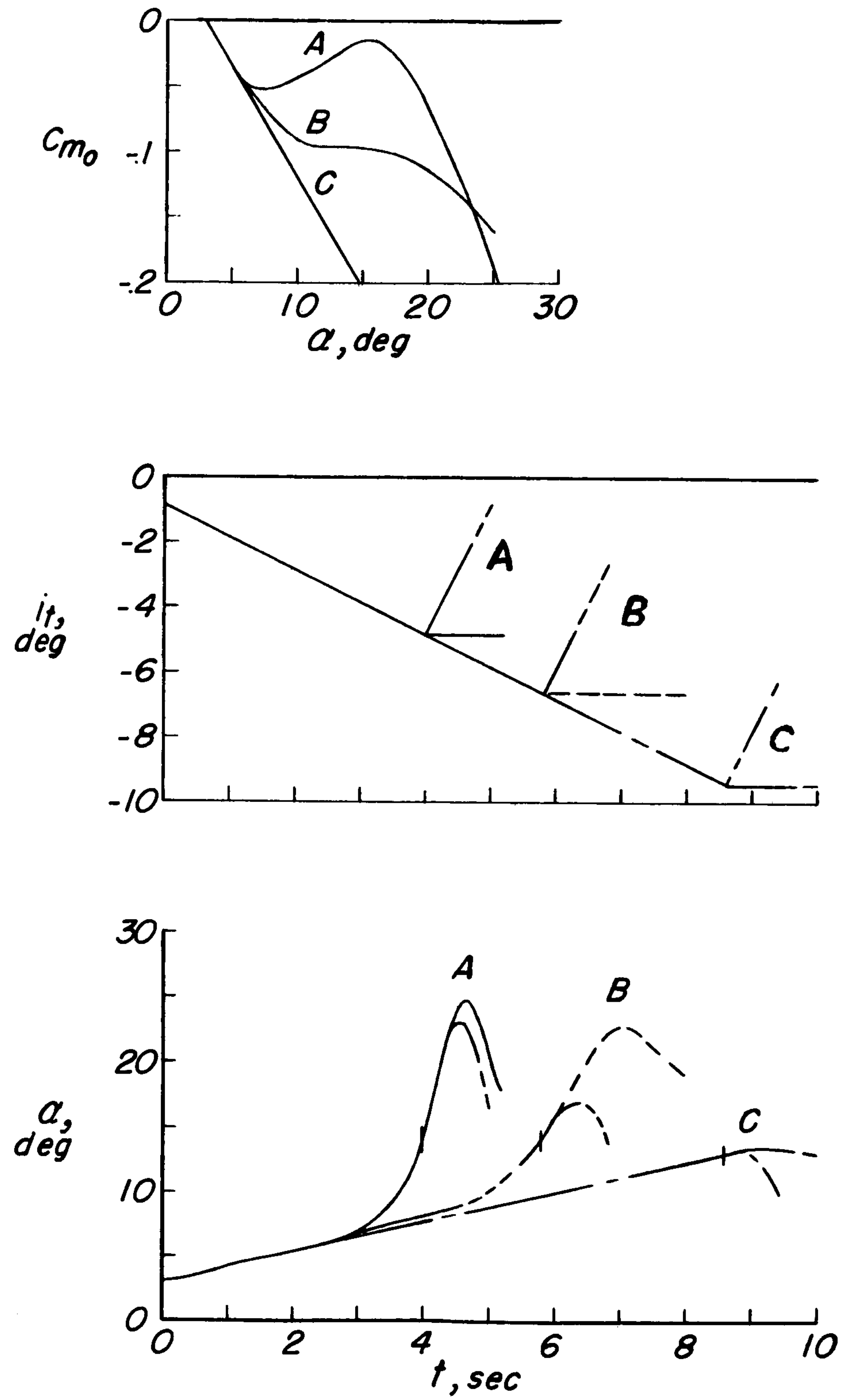

Figure 8.- The interrelated effect of pitching-moment shape and corrective-control motion on pitch-up behavior.

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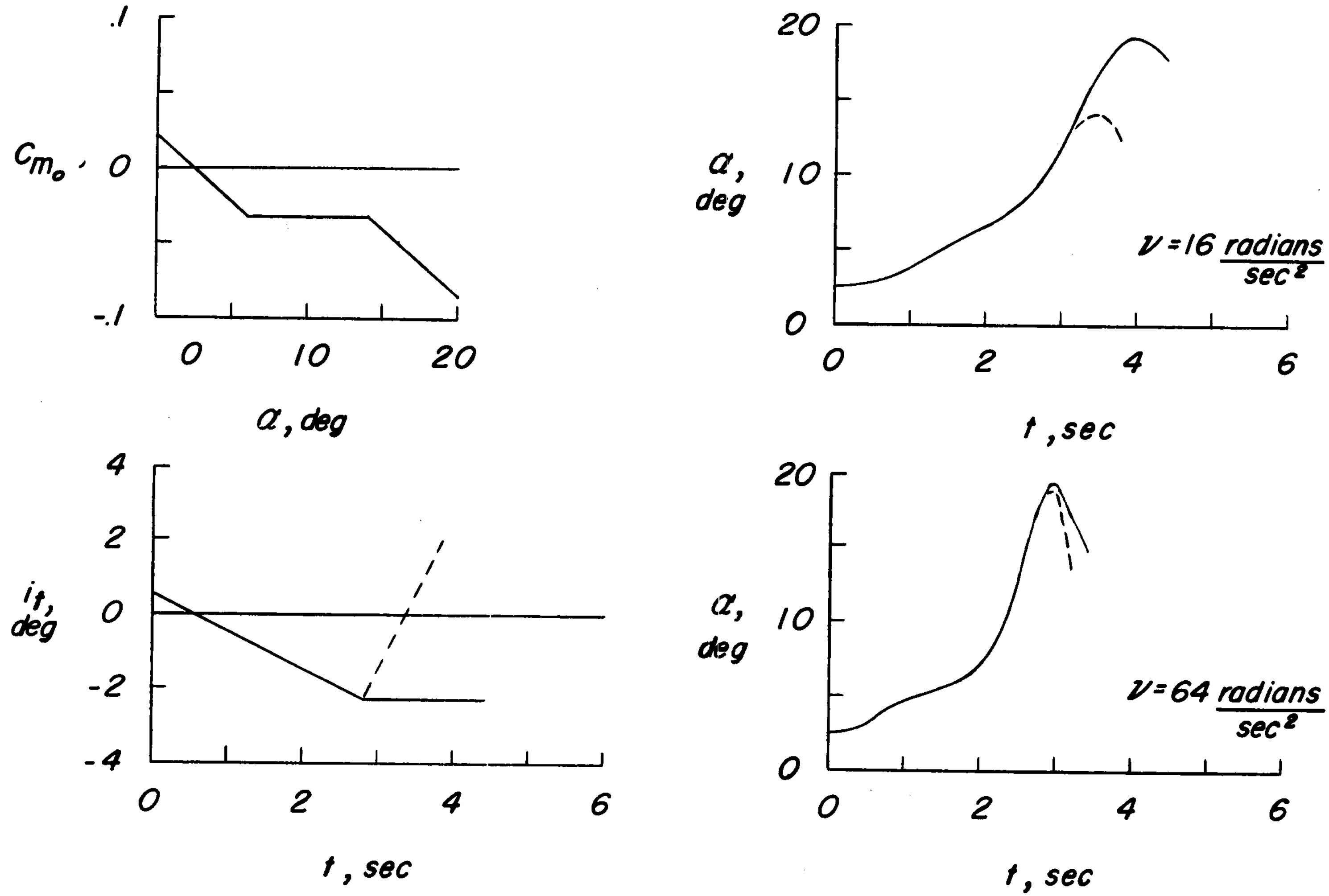

Figure 9.- Effect of dynamic-response parameter $\nu = \frac{\rho V_1{}^2 S\bar{c}}{2I_y}$ on pitch-up behavior.

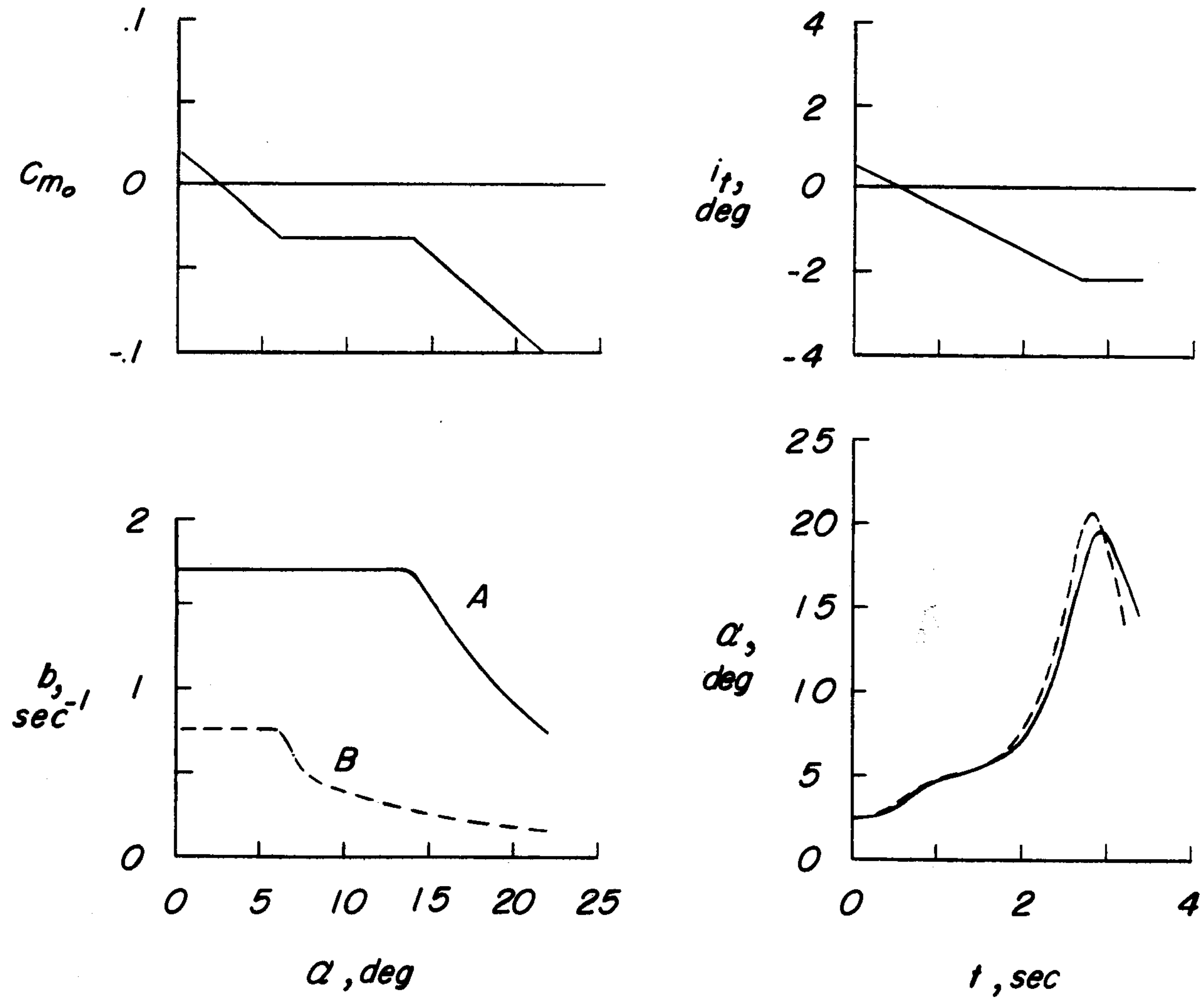

Figure 10.- Effect of damping parameter b on pitch-up behavior. $\nu = 64\ \frac{\text{radians}}{\text{sec}^2}$.

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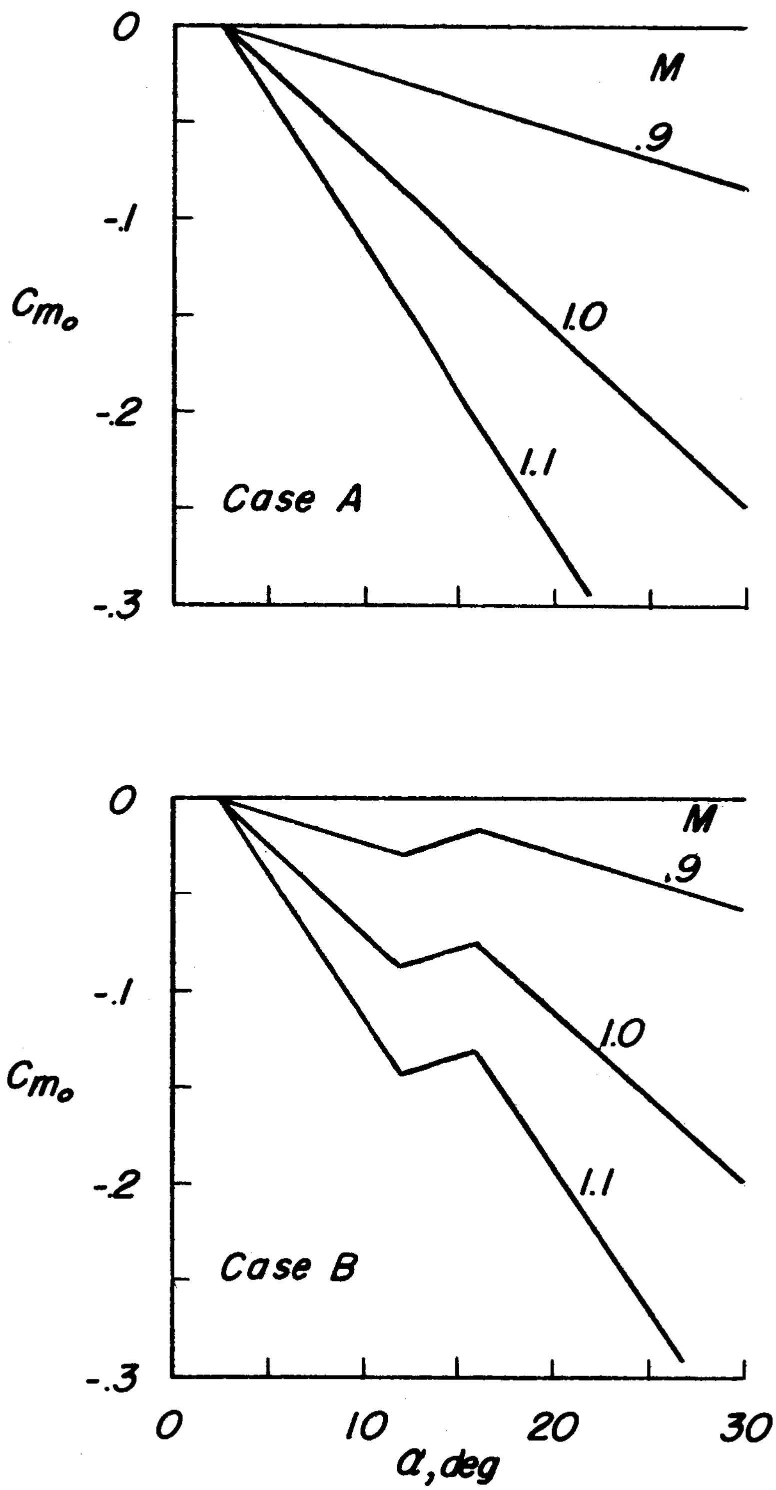

Figure 11.- Static pitching moments used for calculating time histories of figure 12.

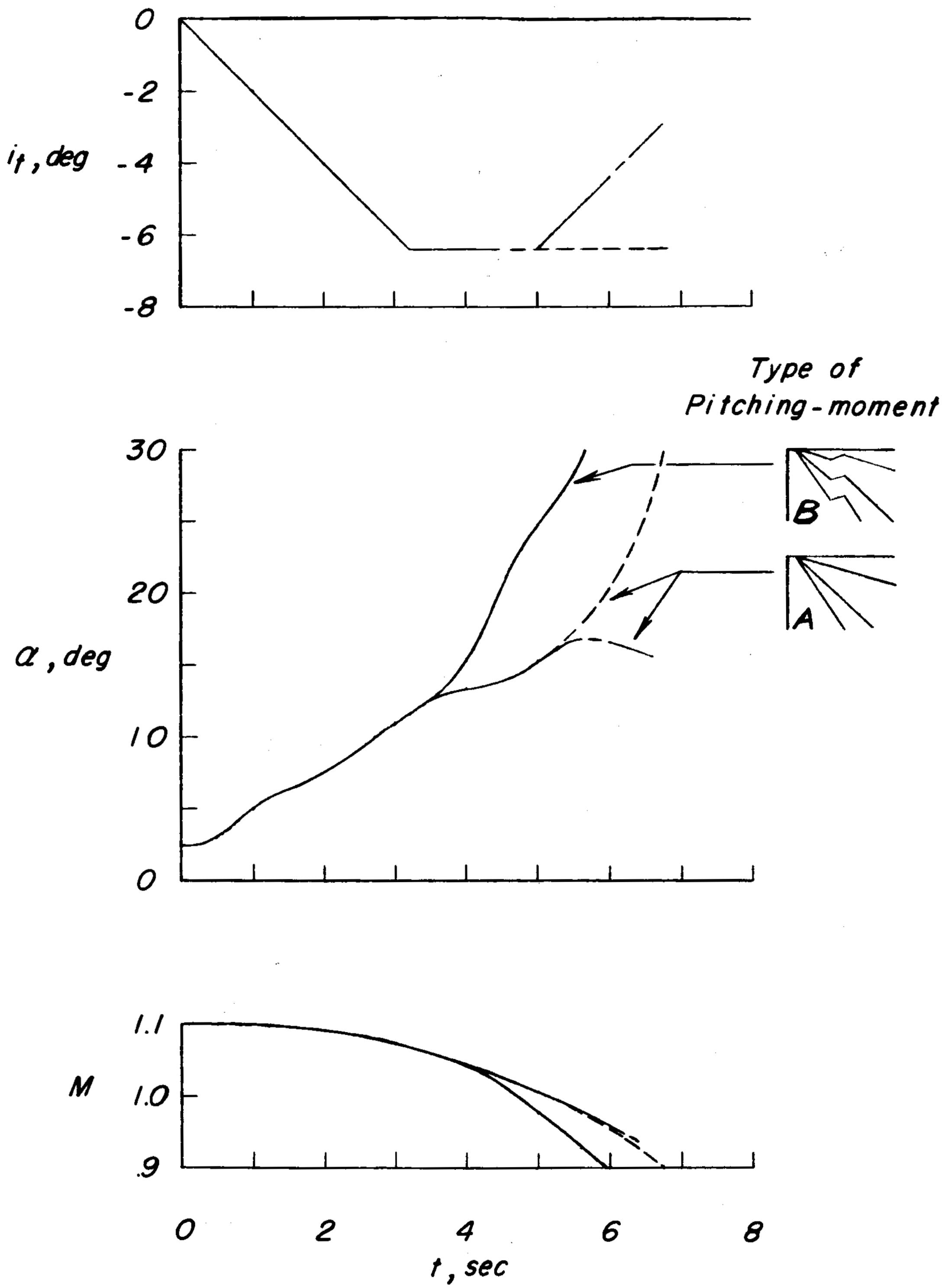

Figure 12.- Effect of a transonic aerodynamic-center shift on airpla: e dynamic response characteristics. $\nu = 19.2 \ \frac{\text{radian}}{\text{sec}^2}$.

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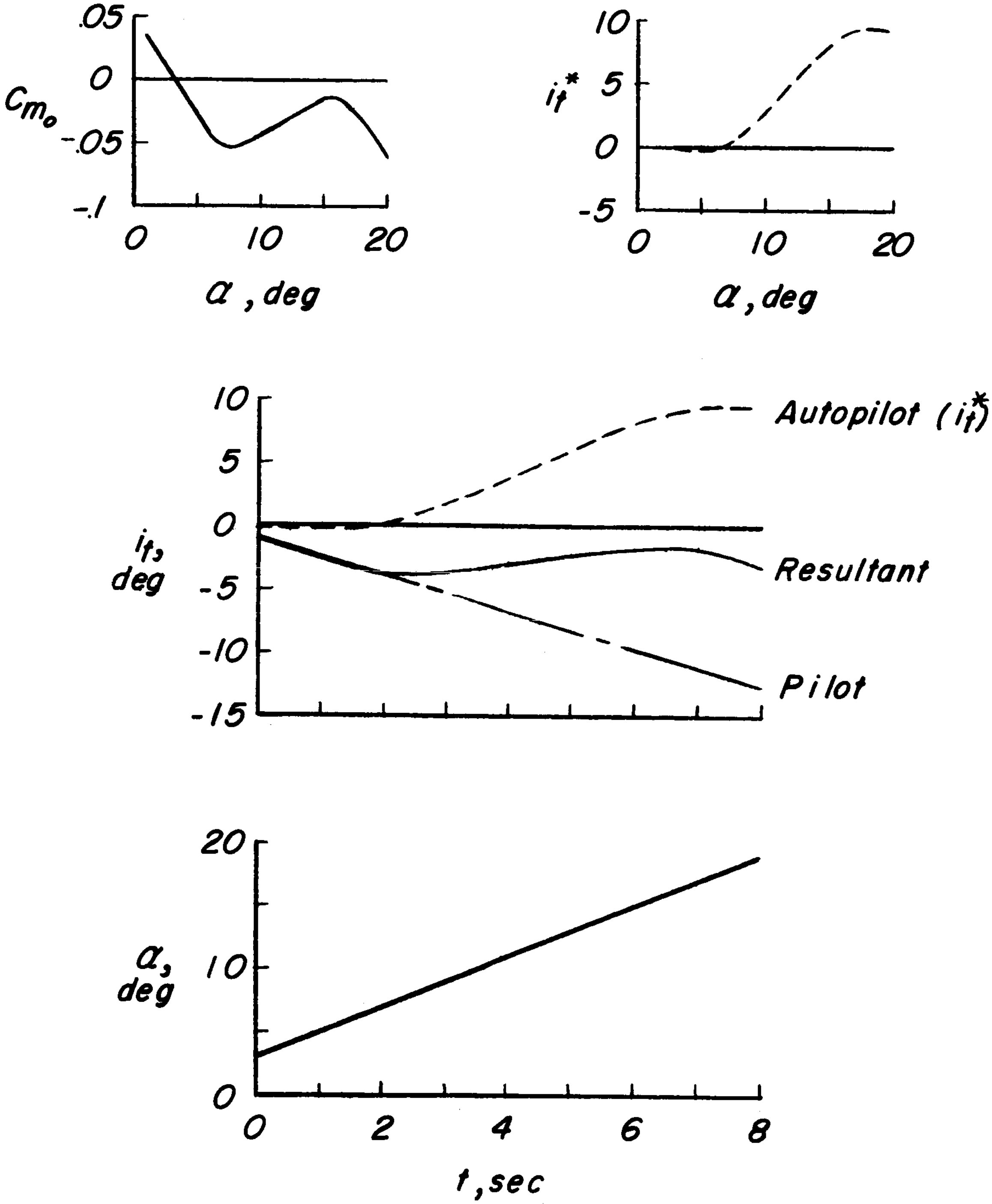

Figure 13.- Autopilot characteristics required to produce linear dynamic response characteristics.

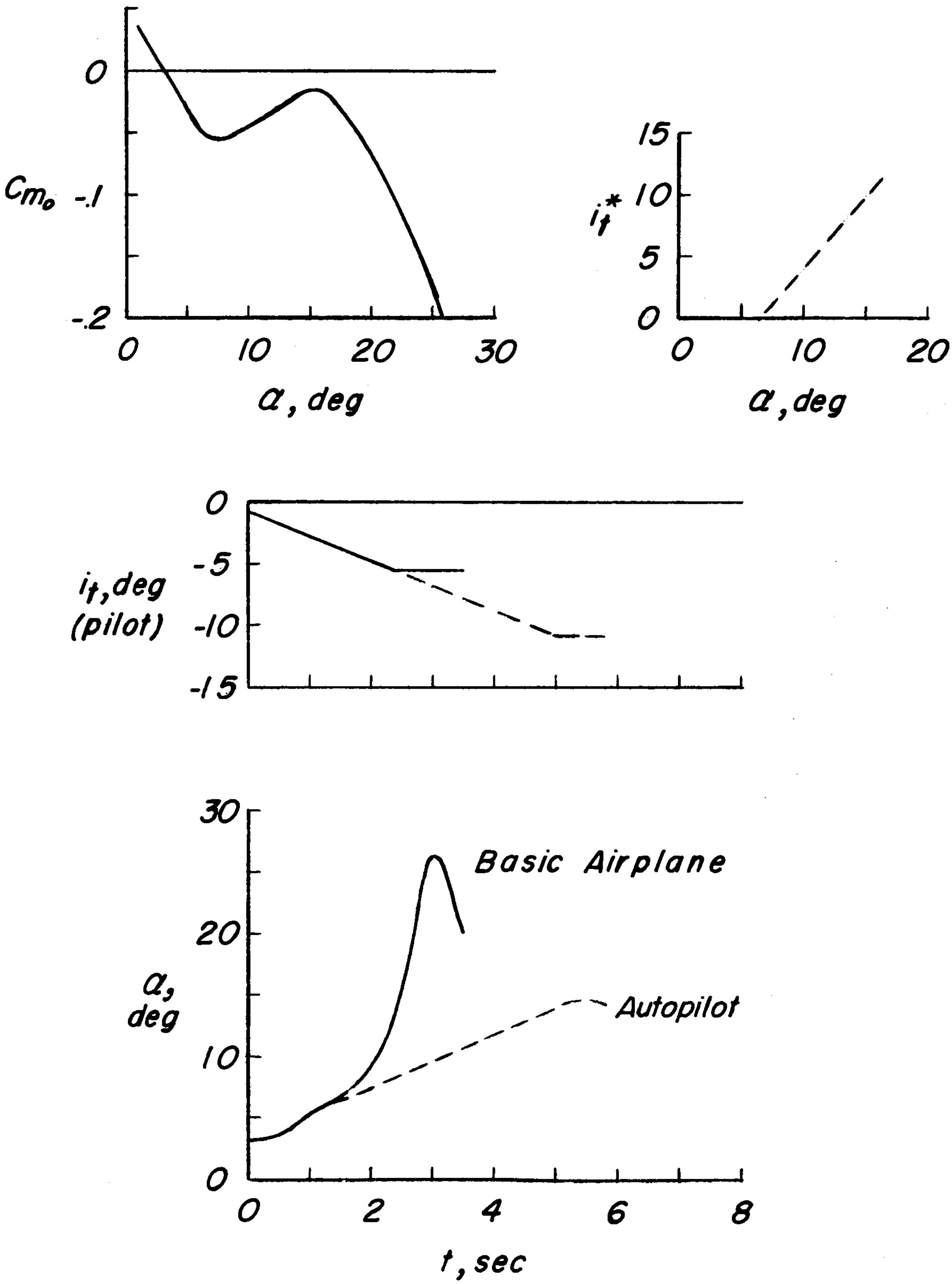

Figure 14.- Effectiveness of an autopilot sensitive to angle of attack in controlling pitch-up.

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